for critical pressure is

$$p_{cr} = \frac{4E}{[12(1-\mu^2)]^{1/2}}\left(\frac{t_s}{R}\right)^2\left\{\frac{\left[\frac{1+(A/bt_s)}{1+(1-\mu^2)A/bt_s}+\frac{A}{bt_s}\right]\left[1+\frac{12(1-\mu^2)I}{bt_s^3}+\frac{12(1-\mu^2)(d/t_s)^2}{1-\mu^2+(bt_s/A)}\right]}{2+(1+\mu)A/bt_s}\right\}^{1/2}$$

where I is the actual moment of inertia of the stiffener, and d is the distance from the skin midplane to the stiffener centroid. Equation (19) of the subject paper was used for E/G_3, and D_3/D was assumed negligible relative to unity.

The important differences between this formula and Professor Buchert's are the added term in the denominator necessary to lower the critical pressure to that for asymmetric buckling, and the larger numerical coefficient $4[12(1-\mu^2)]^{-1/2}$, which is the result of using classical theory. Perhaps the experimental results referred to in the previous comment should be correlated to this formula to provide a reduction factor and, if necessary, referred to a more appropriate formula. That is, a paper by Crawford[1] contains a further linear-theory correction to the previous formula that accounts for the asymmetric section geometry of this class of stiffening. It would be even more appropriate to use those results to establish a proper correlation factor between linear theory and experiment. The correction is made by simply adding to the previous equation the term

$$\frac{2EAd}{R^2b}\left\{\frac{1-(2\mu)/[1+(1-\mu^2)A/bt_s]}{2+(1+\mu)A/bt_s}\right\}$$

for stiffening on the convex surface. The term is subtracted for stiffening on the concave surface.

Equation (10) of the subject paper reduces to the following formula for critical pressure for local instability when the stiffeners provide nodes but no torsional restraint:

$$p_{cr} = \frac{3.62\ Et_s^3}{Rb_s^2}\left[1+\frac{3(1-\mu^2)}{\pi^4}\left(\frac{b}{R}\right)^4\left(\frac{R}{t_s}\right)^2\right]$$

For geometric proportions in the vicinity of their optimum value, the second term in this formula for local instability is less than 1% when $p/E < 10$. Equations (16) and (30), which are equal to the previous equation when its second term is neglected, are therefore appropriate as recommended in the paper when the stiffener's torsional stiffness is neglected for conservatism.

References

[1] Crawford, R. F., "Effects of asymmetric stiffening on buckling of shells," AIAA Preprint 65-371 (July 26–29, 1965).

Comments on "In-Plane Vibration of Spinning Disks"

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There are three aspects of the subject note[1] to be commented upon. First, since none of the four references cited by Huston deal with the in-plane vibrations of a spinning disk, the reader may have been left with the impression that no previous work had been done on this problem. The fact is that at least six earlier papers, Refs. 2–7, have dealt with the more complicated problem of the symmetric (and, in some cases, unsymmetric) in-plane vibrations of a spinning elastic disk attached to a hub of nonzero radius which includes, as a limiting case, the freely spinning disk.

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Second, Eqs. (22) and (23) of Ref. 1 are of questionable physical significance, since the relative order of magnitude of the coriolis effects $\zeta R^2\Omega^2/E$ is of the same order of magnitude as the static strain produced in the disk by the centrifugal loading, and Huston's governing linear equations (1) and (2) already neglect terms of this order.

Third, in this writer's opinion, a satisfactory discussion of the effects of rotation on the frequencies of vibration of a spinning disk is yet to be given. For example, depending on whether the seemingly negligible terms $-\Omega^2 u$ and $-\Omega^2 v$ are added or not to the right-hand sides of Eqs. (1) and (2) of Ref. 1, one finds that, as the rotational frequency Ω increases, the lowest torsional mode of a disk attached to a central hub becomes unstable[4,7] or remains stable,[2,3,6] respectively. Thus, were Huston's equations to be applied to this problem, his argument for neglecting these terms as simply being small[8] would have to be modified. It appears that any complete treatment of the problem should include the effects of the initial, static stresses on the vibration. Some discussion on this point may be found in Ref. 6.

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[3] Biezeno, C. B. and Grammel, R., *Engineering Eynamics: Steam Turbines*, translated by E. F. Winter and H. A. Havemann (Blackie and Sons, Ltd., London, 1954), Vol. III.
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[7] Bhuta, P. G. and Jones, J. P., "Symmetric planar vibrations of a rotating disk," J. Acoust. Soc. Am. **35**, 982–989 (1963).
[8] Huston, R. L., "Wave propagation in rotating elastic media," AIAA J. **2**, 575–576 (1964).

Reply by Author to J. G. Simmonds

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Perhaps it is convenient to reply to these statements in order. First, I extend my apologies to J. G. Simmonds and any others I may have overlooked in the references. I chose primarily those works that I needed for the development of the subject note,[1] the purpose being to investigate the effect of the Coriolis acceleration. In much of the work done by others,[2,3] this effect apparently is discarded as being negligible.

Second, a mathematical model can only be expected to predict physical phenomena in view of the assumptions used in

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